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A Study on Generalized Orthogonal Frequency Division Multiplexing

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Abstract

Orthogonal frequency division multiplexing (OFDM) has been utilized as a major transmission technology for 4-th generation mobile communication, since it has merits such as simple implementation using fast Fourier transform (FFT) and inverse FFT (IFFT) and easy channel estimation. It has excessive bandwidth, however, since it uses rectangular pulses in time domain in order to maintain the orthogonality between subcarriers. Although many efforts have been done to solve the problem, they have drawbacks to sacrifice the channel utility when transmitting short packets. We propose the generalized OFDM which is a generalized form of the conventional OFDM and verify that the proposed scheme is superior spectral efficiency to the conventional OFDM without loss of bit error rate (BER) performance.

Keywords: *OFDM, FFT, IFFT, BER, QPSK*

1. INTRODUCTION

In general, because the low pass frequency characteristics of all vectors of an orthogonal matrix are not desired, we are willing to extract desired vector of the matrix and then multiplex the data using them [1-2].

An orthogonal matrix $\boldsymbol{H}$ can be decomposed as

$$\boldsymbol{H} = \boldsymbol{P} + \boldsymbol{E} \tag{1}$$

where $\boldsymbol{H}$, $\boldsymbol{P}$, and $\boldsymbol{E} \in \mathbb{R}^{N \times M}$, $\boldsymbol{H}^T\boldsymbol{H} = \boldsymbol{I}$, $\boldsymbol{P}$ is a multiplexing matrix which plays a role to allocate M data symbols to N subchannels, and $\boldsymbol{E}$ is an error matrix.
At a transmitter, N symbols are multiplexed using non-orthogonal column vector $\boldsymbol{p}_i$ as

$$\boldsymbol{s} = \sum_{i=0}^{M-1} d_i \boldsymbol{p}_i = \boldsymbol{P}\boldsymbol{d} \tag{2}$$

where $\boldsymbol{p} = [p_0 \quad p_1 \quad \cdots \quad p_{N-1}]^T$ and $\boldsymbol{d} = [d_0 \quad d_1 \quad \cdots \quad d_{M-1}]^T$.

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The received signal may be represented as

$$\boldsymbol{r} = \boldsymbol{s} + \boldsymbol{n} \tag{3}$$

where $\boldsymbol{n}$ is an additive white Gaussian noise.
We utilize a pseudoinverse matrix in order to demodulate as

$$\begin{aligned} \boldsymbol{u} &= \boldsymbol{P}^{\dagger}\left(\boldsymbol{s} + \boldsymbol{n}\right) \\ &= \boldsymbol{d} + \boldsymbol{\eta} \end{aligned} \tag{4}$$

where $\boldsymbol{\eta} = \boldsymbol{P}^{\dagger}\boldsymbol{n}$ and the matrix $\boldsymbol{P}^{\dagger}$ can be represented as

$$\boldsymbol{P}^{\dagger} = \left(\boldsymbol{I} - \boldsymbol{H}^{T}\boldsymbol{E}\right)^{-1}\boldsymbol{H}^{T}. \tag{5}$$

The bit error rate performances for quadrature phase shift keying (QPSK) systems is given by

$$p_b = \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{N_o}\frac{M}{tr\left[\left(\boldsymbol{I} - \boldsymbol{H}^{T}\boldsymbol{E}\right)^{-1}\left(\boldsymbol{I} - \boldsymbol{H}^{T}\boldsymbol{E}\right)^{-T}\right]}}\right) \tag{6}$$

where E_b is the energy per bit and $N_o / 2$ is the noise power spectral density. As we know easily that as the error matrix approaches to the null matrix, the BER performance does to that of the ideal QPSK [3-4].

2. GENERALIZED OFDM SYSTEM SUITABLE FOR SPECTRAL EFFICIENT CHANNEL

In general, to make a matrix $\boldsymbol{P}$ to the nearest $\boldsymbol{H}$ is not a simple problem and there is no any matrices with orthogonality in even mathematical literatures. We seek a matrix $\boldsymbol{P}$ by a recursive method which is orthogonal in both time and frequency domains and efficient in the spectral sense. We obtain an orthogonal matrix $\boldsymbol{P}$ satisfying (1) through the following steps.
Step 1: We prepare an initial matrix as

$$\boldsymbol{P}_0 = \frac{1}{\sqrt{2}}\begin{bmatrix} \ddots & & & & & & \\ & 1 & 1 & & & & \\ & & \ddots & & & & \\ & & & 1 & 1 & & \\ 1 & 1 & & & & & \\ & & & & & & \\ 1 & -1 & & & & & \\ & & & 1 & -1 & & \\ & & & & \ddots & & \\ & & & & & 1 & -1 \\ & & & & & & \ddots \end{bmatrix} \tag{7}$$

which is an orthogonal matrix $\boldsymbol{P}_0^T \boldsymbol{P}_0 = \boldsymbol{I}$. For N orthogonal column vectors, we need N+1 elements for each vector, so the initial matrix can be used by truncating in the form of (N+1, N) matrix in order to transmit N symbol data. For simplicity, we use N as odd number so that the final matrix can have real elements.

Step 2: We reconstruct each column vector by inserting L-N-1 zeros as

$$\boldsymbol{x}_{0,i} = \left[p_{0,i}\left(\frac{N-1}{2}\right) \quad \cdots \quad p_{0,i}(N) \quad \overbrace{0 \quad \cdots \quad 0}^{L-N-1} \quad p_{0,i}(0) \quad \cdots \quad p_{0,i}\left(\frac{N-3}{2}\right) \right], \quad i = 0, 1, \cdots, N, \tag{8}$$

where L is the FFT size. Taking the IDFT (inverse discrete Fourier transform) of each column vector, we have the time domain vector as

$$\boldsymbol{y}_{0,i} = IDFT\left(\boldsymbol{x}_{0,i}\right). \tag{9}$$

Step 3: We remove abrupt jump component by subtracting the first element from all elements as

$$\boldsymbol{q}_{0,i} = \boldsymbol{y}_{0,i} - w_{0,i}(0)\boldsymbol{1}. \tag{10}$$

Step 4: By taking the DFT (discrete Fourier transform), we have the corresponding vector in frequency domain as

$$\boldsymbol{u}_{0,i} = \mathrm{DFT}\left(\boldsymbol{q}_{0,i}\right). \tag{11}$$

Reconstructing by choosing N vectors, we have

$$\boldsymbol{v}_{0,i} = \left[u_{0,i}\left(L - \frac{N-1}{2}\right) \quad \cdots \quad u_{0,i}(L) \quad u_{0,i}(0) \quad \cdots \quad u_{0,i}\left(\frac{N-1}{2}\right) \right]. \tag{12}$$

Step 5: We normalize the power of $\boldsymbol{v}_i$ be unity as

$$\boldsymbol{w}_{0,i} = \frac{\boldsymbol{v}_{0,i}}{\sqrt{\boldsymbol{v}_{0,i}^T \boldsymbol{v}_{0,i}}} \tag{13}$$

which is the i-th column vector of $\boldsymbol{W}_0$.

Step 6: We obtain the nearest orthogonal matrix to $\boldsymbol{P}_1$ as

$$\boldsymbol{P}_1 = \boldsymbol{W}_0 \left(\boldsymbol{W}_0^T \boldsymbol{W}_0\right)^{-\frac{1}{2}}. \tag{14}$$

whose column vector $\boldsymbol{q}_{1,i}$ obtained by taking IFFT has more smoothed waveform than $\boldsymbol{q}_{0,i}$. Repeating from

step 2 to step 6 one more time, we have $\boldsymbol{q}_{2,i}$ which has more smoothed waveform vector than $\boldsymbol{q}_{1,i}$. The k-th repeating is stopped when the first element $w_{k,i}(0)$ in (10) are sufficiently near to zeros.
For the G(4, 5)-OFDM, we can obtain two matrices through the proposed algorithm as

$$\boldsymbol{P}_0 = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}, \quad \boldsymbol{P}_1 = \frac{1}{\sqrt{2}}\begin{bmatrix} -0.2764 & 0 & 0.7236 & 1 \\ 0.7236 & 1 & -0.2764 & 0 \\ -0.8944 & 0 & -0.8944 & 0 \\ 0.7236 & -1 & -0.2764 & 0 \\ -0.2764 & 0 & 0.7236 & -1 \end{bmatrix} \tag{15}$$

and the Frobenius norm of the error matrix corresponding to $\boldsymbol{P}_1$ is $\|\boldsymbol{E}_1\|_F = 9.6838\times10^{-16}$. The time domain responses $\boldsymbol{q}_i$'s of the column vector of $\boldsymbol{P}_1$ are shown in Fig.1. The elements of $\boldsymbol{q}_0$ and $\boldsymbol{q}_2$ are real and the ones of $\boldsymbol{q}_1$ and $\boldsymbol{q}_3$ are imaginary, since $\boldsymbol{p}_0$ and $\boldsymbol{p}_2$ which are the first and third column vector of $\boldsymbol{P}_1$ is even functions, and $\boldsymbol{p}_1$ and $\boldsymbol{p}_3$ are odd functions. From the two figures, we know that all pulses start near zeros and end also near zeros. Since there is no any discrete points between start and end, these pulses does not include any excessive high frequencies but have spectral efficiency suitable for digital communication [5-11].

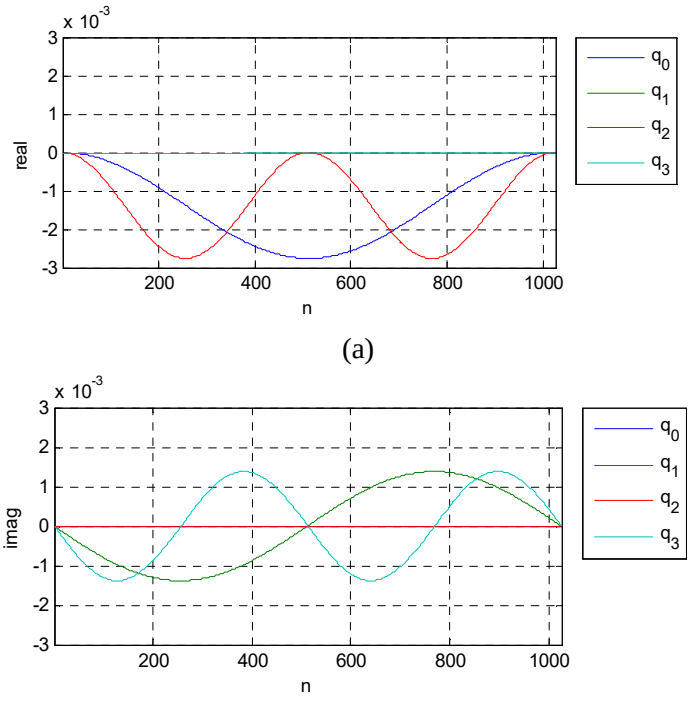


Fig. 1 Time domain pulses ($\boldsymbol{q}_i$, i = 0, 1, 2, 3, $N = 4, L = 1024$), (a) Real part of $\boldsymbol{q}_i$, (b) Imaginary part of $\boldsymbol{q}_i$.

The frequency responses of $\boldsymbol{P}_1$ is depicted in Fig. 2. In the figure, $\boldsymbol{q}_0$ and $\boldsymbol{q}_3$ float near 100 dB, while $\boldsymbol{q}_1$ and $\boldsymbol{q}_2$ are disappeared near 0.2 normalized frequency. The response $\boldsymbol{q}_0$ among the four responses is the nearest one to that of the OFDM and the difference between two responses is 101 dB at the steady state. The noticeable characteristic is that the proposed scheme uses 6% of the spectrum of the conventional OFDM. This is because the proposed scheme uses smooth pulses as shown Fig. 1, while the OFDM uses rectangular pulses

with abrupt jumps in time domain.

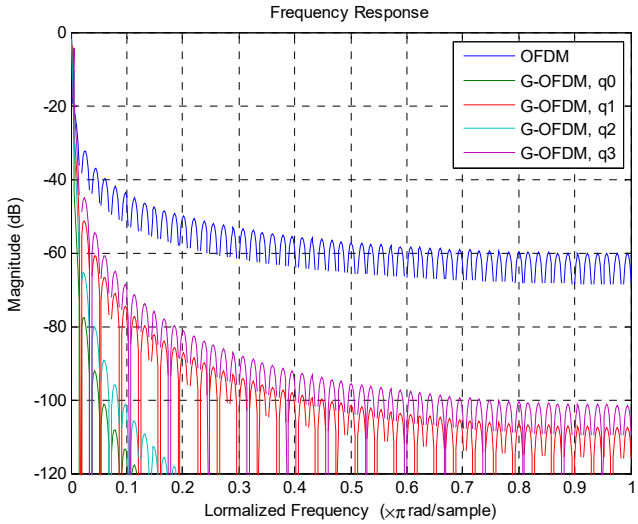


Fig. 2 Frequency responses of prototype filters in frequency domain ($\boldsymbol{q}_i$, i = 0, 1, 2, 3, $N = 4, L = 1024$)

Fig 3 shows the bit error performance of the theoretic QPSK, proposed generalized-OFDM (G-OFDM) and simulation. The analysis given by (6) and simulation agree with the theoretic QPSK. Therefore, it is verified that the proposed scheme has not any loss in view of performance compared with the OFDM and better spectral efficiency.

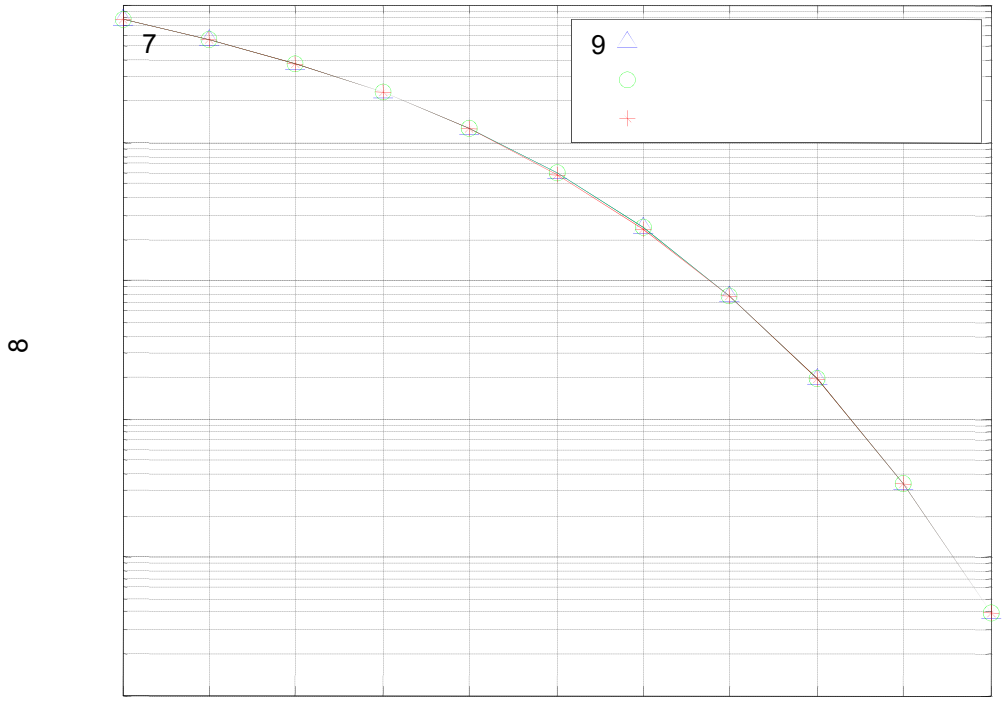


Fig. 3 BER performance of G(4,5)-OFDM, simulation and theoretic QPSK.

3. CONCLUSION

A generalized OFDM scheme was proposed to improve the spectral efficiency without loss of the performance of the conventional OFDM. It was verified that the proposed scheme which used smooth pulses in time domain made through the proposed 6 steps had superior spectral efficiency to the OFDM with abrupt jumps in the pulses, and since the frequency pulses in the proposed scheme are orthogonal, there was no loss in the performance.

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