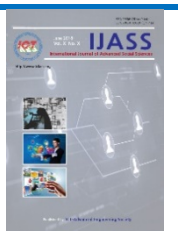
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Time-Varying System with Model Uncertainty Robust State Feedback Linearization for the Nonlinear

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Abstract

In this paper, the discussed the problem of the linearized method such as robustness and disturbance to the nominal nonlinear time-invariant system using the differential geometry. And most recently it dealt with the linearization problem for the time-varying nonlinear system using the extended input-state linearization technique. It discusses the linearization problem for the time-varying nonlinear system with uncertainty. Through the verification of the proposed method, we derive the boundedness of uncertainty.

Keywords: *Nonlinear Time-varying System, Exact Feedback Linearization, Parameter Uncertainty, Boundedness.*

1. Introduction

WSNs consist of hundreds to thousands of low-power multifunctioning sensor nodes, deployed at high density in regions requiring surveillance and monitoring, with limited computational, sensing capabilities and limited power. It is one of important requirements and one of the most challenging problem for many Wireless sensor network applications to prolonged network lifetime [1]. In LEACH, cluster are formed periodically by local coordination and each operation are divided into rounds. LEACHNEW [7] is a new version of LEACH, which modified the threshold of nodes to be cluster head, can avoid the nodes which has low energy to become cluster head.

2. Proposed Algorithm

The physical system is a time-varying parameters expressed by the time-varying nonlinear system with the uncertain model parameter. Many studies on these systems has been in progress [1-6].

The main objective of this paper is to provide an explicit and simple stabilizing controller for single-input single output nonlinear systems using the geometric nonlinear control for the exact feedback linearization and approximate feedback linearization. Consider a standard smooth nonlinear control system affine in the input u(t)

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given by $\dot{x}(t) = f(x(t), t) + g(x(t), t)u(t)$. And, let's define the following vectors - $X = [x^T, t]^T, F = [f^T, 1]^T, G = [g^T, 0]^T$. Using the vector and the concept of differential geometry, the system in the form $\dot{x}(t) = f(x(t), t) + g(x(t), t)u(t)$ is said to be input-state linearizable if there exists a region Ω in R^n, such that the following conditions hold [7-8] :

1. The vector field $\{G, ad_F G, \cdots, ad_F^{n-1} G\}$ has rank n , is linearly independents in Ω.
2. The distribution $[ad_F^i G, ad_F^j G] = span\{G, ad_F G, \cdots, ad_F^{n-1} G\}$ is involutive, for $i, j = 0, 1, 2 \cdots, n-2$, where $[ad_F^i G, ad_F^j G]$ indicates Lie-Bracket and ad_F^i is defined as $ad_F^o G = G$ and $ad_F^i G = [G, ad_F^{i-1} G]$.

The transformation T has the following characteristics [7] :

1. Where $x = 0$ and $t = 0$, the transformation T is zero.
2. The $\phi(x, t)\ L_F^1\ \phi(x, t), \cdots, L_F^{n-1}\ \phi(x, t)$ is a function of the state x and time t.
3. The $\phi(x, t)\ L_F^1\ \phi(x, t), \cdots, L_F^{n-1}\ \phi(x, t)$ has a nonsingular Jacobian matrix for the state x and is one to one mapping.
4. $L_F^n \phi(x, t)$ is a function of the state x, time t and input u, inverses to the function of the input u and corresponds to one to one.

Then, the transformation $\phi(x, t)\ L_F^1\ \phi(x, t), \cdots, L_F^{n-1}\ \phi(x, t)$ is the time dependent state transformation not including the input u, the $L_F^n \phi(x, t)$ is the time dependent state transformation including the input u. An input control signal is than proposed as- $u = \alpha(x, t) + \beta(x, t)v = \frac{-L_F^n \phi(x,t)}{L_G L_F^{n-1} \phi(x,t)} + \frac{1}{L_G L_F^{n-1} \phi(x,t)} v$.

2. The Feedback Linearization For The Nonlinear Time-varying System with The Uncertainty

The proposed method was the extension from the nonlinear time-varying system with the input-state expression of accurate mathematical model system is preferred. However, a mathematical model of the actual control object has omitted parameter and model uncertainty to simplify the expression. Thus, the feedback linearization problem of the nonlinear time-varying system in model uncertainty is considered. Let's consider the following model uncertainty which has the single input-output time varying nonlinear systems.

$$\dot{\hat{x}}(t) = \hat{f}(x(t), t) + \hat{g}(x(t), t)u(t) \tag{1}$$

Here, the output function h is a smooth scalar field which assumed model uncertainty as not present. Where state x is the vector n, $\hat{f}$ and $\hat{g}$ are the smooth vector field n which shows state characteristic and input characteristic of system, u and y represents each input and output. Consider the formula (1) and the uncertainty of the model corresponding to the system which does not include the nominal nonlinear time-varying system, as shown below.

$$\dot{x}(t) = f(x(t), t) + g(x(t), t)u(t) \tag{2}$$

Defined the nominal system relationship between the formula (1) and (2) as follows.

$$\Delta f(x) = \hat{f}(x(t), t) - f(x(t), t) \tag{3}$$

$$\Delta g(x) = \hat{g}(x(t), t) - g(x(t), t)$$

The verified model uncertainty of the formula (3) is time-invariant factor and assume it comes under the closed bound set Ω. Furthermore, when the initial condition x_0 towards the condition x, $\Delta f(x_0)$ is 0. Consider the first matching condition within the vector field $\Delta g(x)$ that presents input characteristic in model uncertainty as $\exists$ a scalar α s.t. $\Delta g \equiv \alpha g$.

The meaning of the first matching condition is that the uncertain vector field Δg is expressed through the linear combination of arbitrary α and vector field g which is smaller than 1. Subsequently, vector field Δg means the spanned by vector field g. When the nominal nonlinear system is feasible to extend in the input-condition linearization of the formula (2), let's take a look at the condition of the uncertain vector field Δf and Δg for the

formula (1) to get feedback linearization. This condition is summarized in the following sufficient condition.

Theorem :

To satisfy the spanned linear space on model uncertainty nonlinear time-varying system as in the formula (1), the feedback linear of input-state conversion is potential.

$$[\Delta \mathrm{F}, \mathrm{a}d_{F^k} G] \in \mathrm{span}\{\mathrm{G}, \mathrm{a}d_F G, \cdots, ad_{F^{k+1}} G\}, k = 0,1, \cdots, n-3 \quad (4)$$

$$\Delta \mathrm{G} \in \mathrm{span}\{\mathrm{G}\} \quad (5)$$

Proof :

Consider the initial state conversion of nominal system in the formula (2-5-1) which has nonlinear time-varying system on model uncertainty.

$$\hat{T}_1 = T_1 \quad (6)$$

When differentiate the formula (6) by time t is as shown.

$$\hat{T}_2 = \dot{\hat{T}}_1 = \frac{\partial \hat{T}_1}{\partial X}(\hat{F} + \hat{G}\mathrm{u})$$

Here, $\hat{F} = F + \Delta F$ and $\hat{G} = G + \Delta G$. When $\mathrm{X} = [X^T, 1]^T$, $\mathrm{F} = [f^T, 1]^T$, $\mathrm{G} = [g^T, 0]^T$, $\Delta F = [\Delta f^T, 0]^T$, $\Delta G = [\Delta g^T, 0]^T$. And as shown in section, where the feedback linear in the nominal nonlinear time-varying system is potential, $L_G T_1$ is 0. Therefore $\hat{T}_2$, the conversion of the formula (6) is expressed as below.

$$\hat{T}_2 = \frac{\partial \hat{T}_1}{\partial X}(\mathrm{F} + \Delta \mathrm{F} + (\mathrm{G} + \Delta \mathrm{G})\mathrm{u}) = L_F T_1 + L_{\Delta F} T_1 \quad (7)$$

The following formula is instituted when differentiate time t over the independent input u in the formula (7).

$$\hat{T}_3 = \dot{\hat{T}}_2 = \frac{\partial \hat{T}_2}{\partial X}(\hat{F} + \hat{G}\mathrm{u}) = L_{\hat{F}^2} T_1 + L_{\hat{G}} L_{\hat{F}} T_1 u \quad (8)$$

Consider the next derivative operator identical equation like Lie.

$$L_{\hat{G}} L_{\hat{F}} = L_{\hat{F}} L_{\hat{G}} - L_{[\hat{F},\hat{G}]} \quad (9)$$

When using the formula (9), it is shown as the formula (8).

$$\hat{T}_3 = \dot{\hat{T}}_2 = L_{\hat{F}^2} T_1 + (L_{\hat{G}} L_{\hat{F}} T_1 - L_{[\hat{F},\hat{G}]} T_1) u = L_{\hat{F}^2} T_1 - L_{[\hat{F},\hat{G}]} T_1 u \quad (10)$$

And Lie Bracket identical equation using the condition of the formula (5) is as below.

$$[\hat{F}, \hat{G}] = [\mathrm{F}, \mathrm{G}] + [F, \Delta G] + [\Delta F, \Delta G] = (1 + \alpha)[F, G] + (1 + \alpha)[\Delta F, G] \quad (11)$$

Here, α is an arbitrary constant. In other word, due to the condition of feedback linearization on the nominal system and Lie Bracket identical equation (11), $L_{[\hat{F},\hat{G}]} T_1$ is 0 and when substitute that to the conversion $\hat{T}_3$, is shown below.

$$\hat{T}_3 = \dot{\hat{T}}_2 = L_{\hat{F}^2} T_1 - (1 + \alpha)(L_{[F,G]} T_1 + L_{[\Delta F,G]} T_1) u = L_{\hat{F}^2} T_1 - (1 + \alpha) L_{[\Delta F,G]} T_1 u \quad (12)$$

The last term of the formula (12) is 0 as condition $[\Delta F, G]$ of the formula (4) in theorem 1 is under the spanned linear space $\mathrm{span}\{\mathrm{G}, ad_F G\}$

$$\hat{T}_3 = L_{F^2} T_1 + L_F L_{\Delta F} T_1 + L_{\Delta F} L_F T_1 + L_{\Delta F^2} T_1 = L_{\hat{F}^2} T_1 \quad (13)$$

The formula (13) is expressed in condition x and time t function, input u is independent. The below formula is found when conversion $\hat{T}_3$ is differentiate by time t.

$$\hat{T}_4 = \dot{\hat{T}}_3 = \frac{\partial \hat{T}_3}{\partial X}(\hat{F} + \hat{G}\mathrm{u}) = L_{\hat{F}^3} T_1 + L_{\hat{G}} L_{\hat{F}^2} T_1 u \quad (14)$$

When applying Lie derivative operator identical equation to the farthest to the right in the formula (14) is as follows.

$$L_{\hat{G}} L_{\hat{F}^2} T_1 = L_{\hat{G}} L_{\hat{F}^2} T_1 - 2 L_{\hat{F}} L_{[\hat{F},\hat{G}]} T_1 + L_{ad\hat{F}^2 \hat{G}} T_1 = L_{ad_{\hat{F}^2} \hat{G}} T_1 \quad (15)$$

Also, when using continuous Lie Bracket identical equation, the following expression is led.

$$\mathrm{a}d_{\hat{F}^2} G = (1 + \alpha) ad_{F^2} G + (1 + \alpha)[F, [\Delta F, G]] + (1 + \alpha)[\Delta F, ad_F G] + (1 + \alpha) ad_{\Delta F^2} G \quad (16)$$

$$[F, [\Delta F, G]] = [F, c_1 G + c_2 ad_{F^G}] = c_1 [\Delta F, G] + (L_{\Delta F} c_1) G + c_2 [\Delta F, ad_F G] + (L_{\Delta F c_2}) ad_{F^G} \quad (17)$$

$$[\Delta F, [\Delta F, G]] = [\mathrm{F}, c_1 G + c_2 ad_F G] = c_1 [\Delta F, G] + (L_{\Delta F c_1}) G + c_2 [\Delta F, ad_F G] + (L_{\Delta F c_2}) ad_F G \quad (18)$$

Here, c_1, c_2 is the scalar field of x. When above identical equations (16), (17), (18) are applied in conversion $\hat{T}_4$, the expression can be shown as next.

$$\hat{T}_4 = \dot{\hat{T}}_3 = L_{\hat{F}^3}T_1 + (1+\alpha)(L_{[\Delta F, ad_F G]}T_1 + 2c_2 L_{[\Delta F, ad_F G]}T_1)u \tag{19}$$

In addition, as $[\Delta F, ad_F G]$ comes under $\mathrm{span}\{G, ad_F G, ad_{F^2} G\}$ in condition of the formula (4), the last term of the formula (19) becomes 0. So, $L_{ad_{\hat{F}^2}\hat{G}}T_1$ is 0 and the conversion $\hat{T}_4$ of the formula (14) is demonstrated as below.

$$\hat{T}_4 = L_{\hat{F}^3}\hat{T}_1 \tag{20}$$

By continuously performing such as process on time t, the below result is obtained when applying the conditions of Lie derivative operator identical equation, Lie Bracket identical equation and the formula (4).

$$\hat{T}_n = \dot{\hat{T}}_{n-1} = \frac{\partial \hat{T}_{n-1}}{\partial X}\left(\hat{F} + \hat{G}u\right) = L_{\hat{F}^{n-1}}T_1 + L_{\hat{G}}L_{\hat{F}^{n-2}}T_1 u \tag{21}$$

When using Leibniz equation on the last term of the above formula (21),

$$L_{\hat{G}}L_{\hat{F}^{n-2}}T_1 = L_{\hat{G}}L_{\hat{F}^{n-2}}T_1 - (n-2)L_{\hat{F}^{n-3}}L_{ad_{\hat{F}}\hat{G}}T_1 + \frac{(n-2)(n-3)}{2!}L_{\hat{F}^{n-4}}L_{ad_{\hat{F}^2}\hat{G}}T_1 - \frac{(n-2)(n-3)(n-4)}{3!}L_{\hat{F}^{n-5}}L_{ad_{\hat{F}^3}\hat{G}}T_1 + \cdots + (-1)^{n-2}L_{ad_{\hat{F}^{n-2}}\hat{G}}T_1 \tag{22}$$

In the previous process, $L_{\hat{G}}T_1 = L_{ad_{\hat{F}}\hat{G}}T_1 = L_{ad_{\hat{F}^2}\hat{G}}T_1 = \cdots = L_{ad_{\hat{F}^{n-3}}\hat{G}}T_1 = 0$. So the formula (21) is expressed as follows.

$$L_{\hat{G}}L_{\hat{F}^{n-2}}T_1 = (-1)^{n-2}L_{ad_{\hat{F}^{n-2}}\hat{G}}T_1 \tag{23}$$

Besides, when applying Lie Bracket operation according to the continuous Lie Bracket $ad_{\hat{F}^{n-2}}\hat{G}$d,

$$L_{\hat{G}}L_{\hat{F}^{n-2}}T_1 = (-1)^{n-2}L_{ad_{\hat{F}^{n-2}}\hat{G}}T_1 = (-1)^{n-2}c_{n-2}L_{\left[\Delta F, ad_{F^{n-3}G}\right]}T_1 \tag{24}$$

Here, c_{n-2} is the scalar function of condition x. Moreover, due to the formula (4), $\left[\Delta F, ad_{F^{n-3}G}\right]$ is under $\mathrm{span}\{G, ad_F G, \cdots, ad_{F^{n-2}}G\}$ which makes the formula (24) to 0.

Hence, the conversion $\hat{T}_n$ is as below.

$$\hat{T}_n = L_{\hat{F}^{n-1}}T_1 \tag{25}$$

Over the previous processes, time-dependent state conversion $\hat{T}_1, \hat{T}_2, \cdots, \hat{T}_n$ is the function of condition x and time t but not a function for input u.

At this stage to get the input conversion, differentiate the formula (25) by time t.

$$v' = \dot{\hat{T}}_n = \frac{\partial \hat{T}_n}{\partial X}\left(\hat{F} + \hat{G}u\right) = L_{\hat{F}^n}T_1 + L_{\hat{G}}L_{\hat{F}^{n-1}}T_1 u \tag{26}$$

$L_{\hat{G}}L_{\hat{F}^{n-1}}T_1$, the relation term of input u in the formula (26) is not 0 as $L_G L_{F^{n-1}}T_1$ is not 0 in the nominal nonlinear time-varying system. So, formula (26) is dependent on the input u and the inverse function exists. The formula (26) can be considered a new external input.

$$v' = L_{\hat{F}^n}T_1 + L_{\hat{G}}L_{\hat{F}^{n-1}}T_1 u \tag{27}$$

Thus, the input conversion u from the formula (27) is demonstrated as follows.

$$u = \hat{a}(x,t) + \hat{\beta}(x,t)\, v' \tag{28}$$

At this point, $\hat{a}(x,t)$ and $\hat{\beta}(x,t)$ is expressed as below.

$$\hat{a}(x,t) = \frac{L_{\hat{F}^n}T_1}{L_{\hat{G}}L_{\hat{F}^{n-1}}T_1} \tag{29}$$

$$\hat{\beta}(x,t) = \frac{1}{L_{\hat{G}}L_{\hat{F}^{n-1}}T_1} \tag{30}$$

When the theorem 1 satisfy the condition as discussed above, the following linearization is occurred according to the conversion of input in the formula (28) and the time-dependent state conversion $\hat{z} = \hat{T}(x,t) = [T_1, L_{\hat{F}}T_1, L_{\hat{F}^2}T_1, \cdots, L_{\hat{F}^{n-1}}T_1]^T$ in nonlinear time-varying system on model uncertainty.

$$\dot{\hat{z}} = A\hat{z} + Bv' \tag{31}$$

(A, B) is representing the Brunovsky matrix. The proof of the theorem1 is completed.

Q.E.D.

(Example)[7]

Consider the nominal time-varying non-linear system, as shown below.

$$\dot{x}(t) = f(x(t),t) + g(x(t),t)u(t) = \begin{pmatrix} x_1 + x_1^2 \\ x_2 \end{pmatrix} + \begin{pmatrix} 1 \\ -t \end{pmatrix} u$$

The state vector $F(x,t)$ and input vector $G(x,t)$ is as bellows :

$$F(x,t) = \begin{bmatrix} x_1 + x_1^2 \\ x_2 \\ 1 \end{bmatrix},\ G(x,t) = \begin{bmatrix} 1 \\ -t \\ 0 \end{bmatrix}$$

As it is a second-order system, the vector field for verifying the controllability will be formed by$\{G, ad_F G\}$, where,

$$G = \begin{bmatrix} 1 \\ -t \\ 0 \end{bmatrix},\ ad_F G = \begin{bmatrix} 1+2x_1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ -t \\ 0 \end{bmatrix} = \begin{bmatrix} 1+2x_1 \\ -t \\ 0 \end{bmatrix}$$

Hence, the vector field $\{G, ad_F G\}$ will be given by $\{G, ad_F G\} = \begin{bmatrix} 1 & 1+2x_1 \\ -t & -t \\ 0 & 0 \end{bmatrix}$.

Therefore, in order to satisfy the above conditions, to be chosen the following equation.

$T_1 = tx_1 + x_2$

We use to compute the second component of the new state vector T_2.

$$T_2 = \dot{T_1} = L_F T_1 = \begin{bmatrix} \frac{\partial T_1}{\partial x_1} & \frac{\partial T_1}{\partial x_2} & \frac{\partial T_1}{\partial t} \end{bmatrix}\begin{bmatrix} x_1 + x_1^2 \\ x_2 \\ 1 \end{bmatrix} = \begin{bmatrix} t & 1 & x_1 \end{bmatrix}\begin{bmatrix} x_1 + x_1^2 \\ x_2 \\ 1 \end{bmatrix} = (t+1)x_1 + tx_1^2 + x_2$$

$$T_3 = \dot{T_2} = L_F T_2 = \begin{bmatrix} \frac{\partial T_2}{\partial x_1} & \frac{\partial T_2}{\partial x_2} & \frac{\partial T_2}{\partial t} \end{bmatrix}\begin{bmatrix} x_1 + x_1^2 \\ x_2 \\ 1 \end{bmatrix} = \begin{bmatrix} t+1+2tx_1 & 1 & x_1 + x_1^2 \end{bmatrix}\begin{bmatrix} x_1 + x_1^2 \\ x_2 \\ 1 \end{bmatrix}$$

$$= (t+2)x_1 + (3t+2)x_1^2 + 2tx_1^3 + x_2 + (1+2tx_1)u$$

And allow the uncertainty $\Delta f(x,t)$ and $\Delta g(x,t)$ as follow.

$$\Delta f(x,t) = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}\ \Delta g = \alpha g = \begin{pmatrix} \alpha \\ -\alpha t \end{pmatrix}$$

In this case, as the theorem 1 is included in condition, the cascade conversion T_1 can be used. And when look at the linear independence condition's inclusive, it is shown as below.

Vector set $\{\hat{G}, ad_{\hat{F}}\hat{G}\} = \begin{pmatrix} \alpha 1 & -2\alpha(1-x_1) \\ -\alpha t & \alpha(2t-1) \\ 0 & 0 \end{pmatrix}$ is the independent linear according to an initial condition. The distribution $\{\hat{G}\}$ is also inclusive. Consequently, the below is the time cascade condition conversion of the nonlinear time-varying system that includes the uncertainty $\Delta f(x)$ and $\Delta g(x)$.

$$\hat{T_1} = tx_1 + x_2 = T_1$$

$$\hat{T_2} = \dot{\hat{T}_1} = x_1 + tx_1 + x_2 = x_1 + t(2x_1 + 2{x_1}^2 + \alpha u) + (2x_2 - \alpha t u)$$

$$= (1+2t)x_1 + t{x_1}^2 + 2x_2 = L_F T_1 + L\Delta_F T_1 = L_{\hat{F}} T_1$$

$$\hat{T_3} = \dot{\hat{T}_2} = < dT_2, f + gu > = (2x_1 + {x_2}^2)(2 + 2t + 2tx_1) + 4x_2$$

$$+ \alpha u(1 + 2tx_1) = {L^2}_{\hat{F}} T_1 + L_{\hat{G}} L_{\hat{F}} T_1 u$$

When considering the conversion $\hat{T_3}$ to the external input v' in the model uncertainty system, it converts to the time-invariant system.

3. Conclusion

In this thesis, the method of input-state linearization for the time-varying nonlinear system with a parameter uncertainty is discussed. To this end, the proposed linearization technique of existing research has been extended to nonlinear systems with the uncertainty. The suggested technique for an extended input-output linearization method is conversed to time-invariant linear system and formulated as time-varying nonlinear system by having multiple integrators. This suggested method presented the necessary and sufficient condition, through the proposed linearization method the bounded for the uncertainty of the parameters were derived and proved. As a result, the linearized system is converted into a time-invariant linear system, it is possible to apply a general linear control techniques.

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